Reliability Estimates (Exponential Distribution)

We know the probability of (or proportion of items) surviving t hours is found by:

$$\hat{R}(t) = e^{-t/\theta} \qquad (8.18)$$

The confidence interval on R(t) is

$$(e^{-t/\hat{\theta}_L} < R(t) < e^{-t/\hat{\theta}_U})$$

where

$\hat{\theta}_L$ and $\hat{\theta}_U$ are the lower and upper confidence limits on $\hat{\theta}$.

Example #3. Based on the data of Example 1, (1) what is the probability of an item's surviving 100 hours? (2) what are the two-sided 95% confidence limits on this probability?

(1) From Equation (8.18)

$$\hat{R}(100) = e^{-100/\hat{\theta}} = e^{-100/160} = 0.535$$

(2) The two-sided confidence limits =

$$(e^{-100/93.65}, e^{-100/333.65})$$

$$= (0.344, 0.741)$$

8.3.2.5.3 CONFIDENCE-INTERVAL ESTIMATES FOR THE BINOMINAL DISTRIBUTION

For situations where reliability is measured as a ratio of the number of successes to the total number of trials, e.g., one shot items, missiles, etc., the confidence interval is determined by consideration of the binominal distribution. Table XI of Hald's Statistical Tables and Formulas (John Wiley and Sons, Inc., New York, 1952) and Ref. 11 gives 95% and 99% confidence limits for a wide range of values. Figure 8.3.2.5.3-1 allows a rough estimate to be made when the number of successes (S) and the number of trials (N) are known.

Example #4. S = 8; N = 10. (a) What is the reliability estimate? (b) What are the two-sided upper and lower 95% confidence limits? Answers: (a) 0.80; (b) 0.98 and 0.43.

More detailed analyses of confidence limits and intervals, with many more examples under a variety of circumstances, and for a variety of disitributions, e.g., binominal, gamma, Weibull, etc., are given in Refs. 6, 9, 10 and 11.